

# Tailored jump operators for purely dissipative quantum magnetism

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I propose an architecture for the realization of dissipative quantum many-body spin models. The dissipative processes are mediated by interactions with auxiliary particles and lead to a widely tunable class of correlated quantum jump operators. These findings enable the investigation of purely dissipative spin models, where coherent dynamics is entirely absent. I provide a detailed review of a recently introduced variational method to analyze such dissipative quantum many-body systems, and I discuss a specific example in terms of a purely dissipative Heisenberg model, for which I find an additional disordered phase that is not present in the corresponding ground state phase diagram.

## I. INTRODUCTION

Emerging quantum technologies such as quantum simulation or quantum metrology inherently require control over nonclassical correlations such as quantum entanglement. For a long time, a central belief has been that in order to realize such technologies, it would be absolutely necessary to perfectly isolate the quantum system of interest from its environment, as otherwise there would be unavoidable decoherence destroying the nonclassical states. However, as has been demonstrated in a series of landmarking theoretical and experimental works [1–7], it is possible to use carefully engineered couplings to the environment to dissipatively prepare interesting quantum many-body states as the stationary states of the evolution of such an open quantum system.

These initial results have sparked a huge interest in the study of dissipative quantum many-body systems. Besides investigation of the aforementioned dissipative quantum state engineering [8–17], there is also a large body of works investigating their fundamental properties. In particular, identifying and understanding phase transitions in dissipative quantum systems has emerged as a central topic in this context [18–38].

The theoretical analysis of dissipative quantum many-body systems is significantly more challenging than for equilibrium systems due to the lack of an underlying concept corresponding to the partition function. Consequently, the available tools have been quite limited. For one-dimensional problems, methods related to the density matrix renormalization group have been successfully used [25, 26, 39–44], however their extension towards higher dimensional problems remains an outstanding challenge. In higher dimensions, most analyses have been limited to mean-field treatments [21–23, 30, 31, 45–50], however, recent results cast severe doubts on the validity of such a mean-field decoupling [43, 51–53], even in large spatial dimensions. Generic non-equilibrium methods such as the Keldysh formalism can be extended to open systems [29, 54], but are notoriously difficult to

treat in the presence of strong interactions [55]. As a result, there is increasing activity to find novel computational approaches that do not suffer from these limitations [56–58]. One promising route is a variational principle for open quantum systems [59], which has given quantitatively reliable results for a variety of different systems [52, 53, 60, 61], and which appears to correctly describe phase transitions above the upper critical dimension [62].

The physical realizations that are being discussed in the context of dissipative quantum many-body systems are quite diverse. On the one hand, a wide range of works investigates the coupling of atoms or ions to metastable electronic excitations [19, 32, 34, 63]. On the other hand, there is also a very strong activity in the context of solid state systems, e.g., in the context of exciton-polariton condensates [64–66], solid state cavity arrays [45, 67–70], or nitrogen-vacancy defect centers in diamond [71–74]. A key point in all these investigations is that the steady state of the open quantum system is not a thermal state with respect to the system Hamiltonian. One common way to realize this is to consider an explicit time-dependent driving field, e.g., an external laser, which is why such systems are sometimes referred to as being “driven-dissipative”. However, it is important to note that non-Markovian systems offer similar possibilities [75, 76], and hence it is possible to realize interesting dissipative quantum many-body models also without the help of external driving fields.

Nevertheless, it remains a central challenge to realize dissipative processes in a controlled way. In the context of driven-dissipative settings, the dissipation usually arises from coupling the eigenstates of the system to the vacuum of the electromagnetic field, hence the dissipation naturally acts in the eigenbasis of the system. In many cases, however, it would be desirable to engineer other dissipation channels, e.g., for the production of steady state coherence between the eigenstates. While there are proposals to realized largely arbitrary dissipation channels in the framework of universal quantum simulation architectures [3, 15, 77, 78], it is highly desirable to implement tunable dissipation without requiring such a detailed level of control over the setup.

In this paper, I discuss an architecture for the realization of largely arbitrary dissipation channels. The cen-

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tral strategy is to turn coherent interactions with auxiliary particles into effective dissipative elements for the reduced system under consideration. I will discuss the implementation of single-particle and two-particle operators describing dissipative quantum jumps, focussing on the case where the resulting effective dynamics is purely dissipative and coherent dynamics is entirely absent. Crucially, such purely dissipative systems are still fully quantum if the associated quantum jump operators act in different eigenbases [36]. As a specific example, I will focus on dissipative quantum magnetism in the form of a dissipative Heisenberg model. I will analyze the phase diagram of this model using the variational principle for open quantum systems and comment on the differences to the ground state phase diagram.

## II. TAILORING QUANTUM JUMP OPERATORS

Here, we are interested in realizing dissipative spin models for an ensemble of atoms or molecules loaded into an optical lattice with uniform filling. We will be investigating a case where the dissipative elements of the dynamics are mediated by a set of auxiliary atoms, see Fig. 1. The starting point of the analysis is the microscopic quantum master equation in Lindblad form, which includes both the spin ensemble and the auxiliary atoms, given by

$$\frac{d}{dt}\rho = -i[H, \rho] + \sum_i \mathcal{D}(c_i), \quad (1)$$

where  $\rho$  is the density operator describing the state of the system,  $H$  is the Hamiltonian responsible for the coherent part of the dynamics, and the dissipative terms are given in terms of quantum jump operators  $c_i$  by

$$\mathcal{D}(c_i) = c_i \rho c_i^\dagger - \frac{1}{2} (c_i^\dagger c_i \rho + \rho c_i^\dagger c_i). \quad (2)$$

In the following, we will obtain a reduced description of the dynamics only for the atoms in the spin ensemble.

### A. Effective operator formalism

The general setup we are considering has dissipative terms only within the auxiliary atoms. Therefore, we aim for an effective description where the dynamics of the auxiliary atoms is integrated out. To be specific, we assume that the auxiliary atoms are two-level systems with one state decaying into the other; a straightforward realization is optical pumping by laser coupling one of the two levels to a fast decaying electronic excitation, see Fig. 1. Within this assumption, we will employ the effective operator formalism [79] to derive an effective quantum master equation for the spin ensemble.

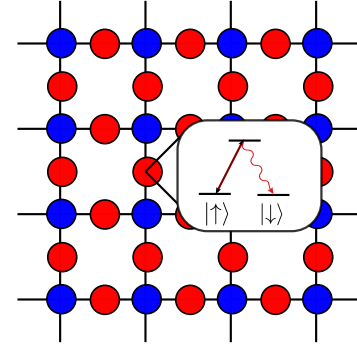


FIG. 1. Setup of the system where atoms of the spin ensemble (blue) are interspersed with auxiliary atoms (red) needed to mediate dissipative couplings. The auxiliary atoms have one spin state decaying into the other by means of optical pumping via an electronically excited state.

Within the effective operator formalism, the resulting master equation is given by

$$\dot{\rho} = -i[H^{\text{eff}}, \rho] + \sum_k \mathcal{D}(c_k^{\text{eff}}), \quad (3)$$

with the effective Hamiltonian  $H_{\text{eff}}$  and effective jump operators  $c_{\text{eff}}^k$  being given by

$$H_{\text{eff}} = H_g - \frac{1}{2} \sum_k V_k^- \left[ \tilde{H}_k^{-1} + (\tilde{H}_k^{-1})^\dagger \right] V_k^+ \quad (4)$$

$$c_k^{\text{eff}} = c_k \tilde{H}_k^{-1} V_k^+. \quad (5)$$

Here,  $H_g$  is the Hamiltonian within the ground state manifold, i.e., involving only states that are not subject to dissipation.  $V_k^+$  and  $V_k^-$  are interaction terms that couple an ensemble atom to the decaying state or away from it, respectively. Finally,  $\tilde{H}_k$  is the non-Hermitian Hamiltonian for the  $k$ th auxiliary atom, given by

$$\tilde{H}_k = H_k^e - \frac{i}{2} c_k^\dagger c_k, \quad (6)$$

which  $H_k^e$  being the part of the Hamiltonian that couples only within the manifold of states subject to dissipation. Note that although  $\tilde{H}_k$  is non-Hermitian, the overall effective Hamiltonian  $H^{\text{eff}}$  is always Hermitian.

In the following, we will be interested the case where the effective dynamics is purely dissipative. To this end, we consider situations where  $H_g = 0$ , which can be realized by ensuring that there are no interactions within the spin ensemble, while possible effective terms corresponding to local operators can be canceled using suitable local potentials. The contribution to the effective Hamiltonian of the auxiliary atoms vanishes in case the non-Hermitian Hamiltonian is purely imaginary; this occurs when the two states of the auxiliary atom have the same energy in a suitable rotating frame.

## B. Single-particle jump operators

As a first step, let us discuss the requirements for the realization of arbitrary single-particle jump operators. To be specific, we will exemplify the proposed architecture by considering the realization of the jump operator  $|\neg\chi\rangle\langle+|$ , as this jump operator is not acting within the eigenbasis of the spin ensemble and therefore represents are particularly challenging case.

Without loss of generality, we can assume that the spin up state of the auxiliary atom is decaying into the spin down state, hence the microscopic jump operator is given by  $c_k = \sqrt{\gamma}\sigma_k^- = \sqrt{\gamma}[\sigma_k^x - i\sigma_k^y]/2$  in terms of the Pauli matrices  $\sigma_k^\alpha$ , with  $\gamma$  being the associated decay rate. Then, we will be interested in the case where the interaction Hamiltonian between the  $i$ th atom in the spin ensemble and the  $k$ th auxiliary atoms is given by

$$H_{ik} = E_0|\neg\chi\rangle\langle+|_i\sigma_k^+ + \text{H.c.} = \frac{E_0}{2} [\sigma_i^z\sigma_k^x + \sigma_i^y\sigma_k^y], \quad (7)$$

with  $E_0$  being an energy scale related to the concrete physical implementation of the interaction. Such tunable spin interactions can be efficiently realized in a wide range of physical systems, most notably Rydberg-dressed atoms [80, 81] and rotational excitations of ultracold polar molecules [82–84]. Within the context of the effective operator formalism, we find that the term containing  $\sigma_k^+$  corresponds to  $V_k^+$ , while its Hermitian conjugate is identical to  $V_k^-$ . Then, we find for the effective jump operator

$$c_i^{\text{eff}} = \frac{E_0}{\sqrt{\gamma}}|\neg\chi\rangle\langle+|_i|\downarrow\rangle\langle\downarrow|_k. \quad (8)$$

In leading order in  $E_0/\gamma$ , the auxiliary atoms are confined to the spin down state and hence the auxiliary atom can be factorized out from the dynamics, leaving only the desired jump operator  $|\neg\chi\rangle\langle+|$ .

## C. Multi-particle jump operators

Now, let us turn to the realization of two-body jump operators. In the same way as the realization of single-particle jump operators requires two-body interactions including one of the auxiliary atoms, two-body jump operators require the presence of three-body interactions. Such three-body terms can arise in a variety of different contexts, e.g.: (i) Strong blockade effects in Rydberg-dressed atoms lead to higher-order interaction terms [85]. (ii) Canceling the two-body interaction leave three-body interactions as the leading interaction term [86]. (iii) Time-dependent driving of a system can lead to three-body interactions in the rotating frame of the driving [87]. In the following, we will not make detailed assumption about the underlying physical mechanism to realize the three-body interaction terms, allowing to retain a general treatment of the emerging dissipative many-body dynamics.

As an example, let us study the implementation of a correlated jump operator of the form  $|\psi_+\rangle\langle\psi_-|$ , where the states  $|\psi_\pm\rangle$  are the Bell states

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle). \quad (9)$$

As this jump operator pumps the system towards the maximally entangled state  $|\psi_+\rangle$ , it can be seen as a building block for the efficient generation of entangled quantum states using dissipation. In close analogy to the preceding subsection, we can realize such a jump operator with the help of the auxiliary atoms. Specifically, we find this jump operator being realized when the interaction Hamiltonian  $H_{ijk}$  between two atoms of the spin ensemble  $i$  and  $j$  and one auxiliary atom  $k$  takes the form

$$\begin{aligned} H_{ijk} &= E_0|\psi_+\rangle\langle\psi_-|_k\sigma_k^+ + \text{H.c.} \\ &= \frac{E_0}{4} [\sigma_i^x\sigma_j^x\sigma_k^x + \sigma_i^y\sigma_j^y\sigma_k^x - \sigma_i^z\sigma_j^z\sigma_k^x + \sigma_k^x]. \end{aligned} \quad (10)$$

Calculation of the effective jump operators then leads to the desired result.

## III. VARIATIONAL ANALYSIS OF PURELY DISSIPATIVE HEISENBERG MODELS

We will now turn to the analysis of purely dissipative spin models that can be realized using the techniques laid out in the previous section. However, before turning to a detailed analysis of the models, let us go through a detailed introduction to the variational principle presented in [59], which will serve as the basis for our investigation.

### A. Introduction to the variational principle for dissipative quantum many-body systems

In general, finding the exact solution to the steady state of the quantum master equation is an exponentially hard problem. Therefore, we will aim for an approximate solution that can be calculated efficiently. To this end, we perform a variational parametrization of the density matrix. As the first step, we take a variational basis in terms of tensor products of single atom density matrices. As a further simplification, we assume that all single atom density matrices are identical. Then, we have

$$\rho = \rho_0 \otimes \rho_0 \otimes \cdots \quad (11)$$

$$\rho_0 = \frac{1}{2} (1 + \alpha_x\sigma_x + \alpha_y\sigma_y + \alpha_z\sigma_z). \quad (12)$$

Here,  $\alpha_x$ ,  $\alpha_y$ , and  $\alpha_z$  are our (real) variational parameters, which we will vary to find the best possible approximation to the steady state. Consequently, we have boiled down the full quantum many-body problem down to the computation of three variational parameters. Note that we have already incorporated the requirement of the

trace being identical to one into the construction of  $\rho_0$ , as the Pauli matrices are traceless. However, the constraint that the density matrix must be non-negative leads to the condition  $\alpha_x^2 + \alpha_y^2 + \alpha_z^2 \leq 1$ , which has to be accounted for within the numerical optimization.

As the underlying variational principle, we want to minimize a suitable norm  $\|d/dt\rho\|$ , as the norm going to zero indicates that we have found the true steady state. Here, the suitable norm is given by the trace norm, i.e.,  $\text{Tr}\{|d/dt\rho|\}$  [59]. However, computing the variational norm is still an exponentially hard task, even for our rather simple ansatz discussed above. Therefore, we minimize a slightly different variational functional that provides an upper bound. In the case of our ansatz, this upper bound can be given as

$$\left\|\frac{d}{dt}\rho\right\| \leq \sum_{\langle ij \rangle} \left\|\frac{d}{dt}\rho^{(ij)}\right\|, \quad (13)$$

where  $d/dt\rho^{(ij)}$  is the time derivative of the reduced system consisting of only two atoms [59]. For system sizes where a direct comparison can be made, taking the upper bound instead of the full problem leads to very similar results [52]. Crucially, the reduced time derivative is given by a  $4 \times 4$  matrix, which can be easily diagonalized. In the case of a homogeneous system, all terms of the sum are identical, meaning that we have to minimize the norm of a single  $4 \times 4$  matrix.

In the most general case, the contribution to the operator  $d/dt\rho^{(ij)}$  consists of three different parts,

$$\frac{d}{dt}\rho^{(ij)} = \dot{\rho}_{loc}^{(ij)} + \dot{\rho}_{int}^{(ij)} + \dot{\rho}_{MF}^{(ij)}. \quad (14)$$

The first term,  $\dot{\rho}_{loc}^{(ij)}$ , refers to terms that act only within the local Hilbert space of a single site. The second term,  $\dot{\rho}_{int}^{(ij)}$  refers to two-particle operators describing coherent interactions and correlated jump operators that act only on particles  $i$  and  $j$ . These first two terms are identical to the ones found in the microscopic quantum master equation. The third term,  $\dot{\rho}_{MF}^{(ij)}$  is slightly more subtle and corresponds to a mean-field treatment of the sites surrounding  $i$  and  $j$ , given by

$$\dot{\rho}_{MF}^{(ij)} = \sum_k \text{Tr}_k \left\{ \left[ H_{ik} + H_{jk}, \rho^{(ijk)} \right] + \mathcal{D}(c_{ik}) + \mathcal{D}(c_{jk}) \right\}, \quad (15)$$

where  $\rho^{(ijk)} = \rho^{(i)} \otimes \rho^{(j)} \otimes \rho^{(k)}$  is the reduced density operator for three sites. If we compose the interaction term  $H_{ik}$  into operators of the form  $H_{ik} = A_i \otimes B_k$ , we can write their contribution to  $\dot{\rho}_{MF}^{(ij)}$  as

$$\text{Tr}_k \left\{ \left[ H_{ik}, \rho^{(ijk)} \right] \right\} = \text{Tr} \{ B_k \rho_k \} \left[ A_i \otimes 1_j, \rho^{(ij)} \right], \quad (16)$$

which describes a mean-field decoupling of the interaction [88]. Similar expressions can also be found in the case of correlated jump operators acting on multiple sites at the same time.

The sum over  $k$  in Eq. (15) runs over all sites adjacent to  $i$  and  $j$  and therefore contains  $2(z-1)$  terms, where  $z$  is the coordination number of the underlying lattice. Crucially, this results in the mean-field term being dominant in the limit of large  $z$ . If we want to capture, for instance, the changes in phase diagrams with changing  $z$ , it is therefore convenient to renormalize all coupling constants  $\lambda$  corresponding to two-particle operators (both coherent and dissipative) according to  $\lambda = \lambda'/(z-1)$ . Then, the mean-field term remains constant in the large  $z$  limit. However, this does not apply to the second term  $\dot{\rho}_{int}^{(ij)}$ , which decays like  $1/(z-1)$  in this limit. As this term captures the buildup of correlations, this means that correlations become irrelevant in the limit of large  $z$  and the product state solution becomes exact.

The variational approach has proven to be especially successful for the analysis of dissipative phase transitions [62]. The unique advantage is the possibility to interpret the variational norm in close correspondence to the free energy functional for equilibrium systems. Then, in close analogy to Landau theory for equilibrium transitions, one can formally perform a series expansion of the variational norm in the order parameter  $\phi$  of the dissipative phase transition. A common situation is that due to symmetry reasons, the odd powers of the expansion vanish and one is left with a  $\phi^4$  theory according to

$$\|\dot{\rho}^{(ij)}\| = u_0 + u_2\phi^2 + u_4\phi^4 + O(\phi^6). \quad (17)$$

Such a  $\phi^4$  theory has a phase transition at a critical value of  $u_2 = 0$ , going from a disordered phase with  $\phi = 0$  for  $u_2 > 0$  to an ordered phase with  $\phi \neq 0$  for  $u_2 < 0$ . Close to the phase transition, the order parameter behaves according to  $\phi \sim u_2^{1/2}$ , from which we can identify the critical exponent  $\beta = 1/2$ .

## B. Application to purely dissipative Heisenberg models

The Heisenberg model is a paradigmatic model for the study of quantum magnetism. Here, we will be interested in a particular variant of a spin 1/2 model, where the coupling constant along two axes is identical, which is also known as the XXZ model. Then, the Hamiltonian of the equilibrium model is given by

$$H = -J \sum_{\langle ij \rangle} \sigma_x^{(i)} \sigma_x^{(j)} + \sigma_y^{(i)} \sigma_y^{(j)} + (1 - \lambda) \sigma_z^{(i)} \sigma_z^{(j)}. \quad (18)$$

Constructing a dissipative analog of this model can be done in different ways, but in general, the dissipative model should exhibit the same symmetries as the equilibrium one. This means that for  $\lambda = 0$ , the model exhibits an  $SU(2)$  symmetry corresponding to global rotation around any axis, while for  $\lambda \neq 0$  this symmetry is partially lifted and only a  $U(1)$  symmetry corresponding to rotations around the  $z$  axis remains. Here, we have

chosen the interaction in the  $XY$  plane to be ferromagnetic, however, an underlying mirror symmetry along the  $z$  axis results in the antiferromagnetic case having the same spectrum [89].

The way we construct the dissipative Heisenberg model is to consider two sets of jump operators. The first set of jump operators pumps the system into a dark state where the  $SU(2)$  symmetry is spontaneously broken. In the ferromagnetic case, these jump operators have the form

$$c_{ij} = |\uparrow\uparrow\rangle\langle\psi_-|_{ij} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (19)$$

$$c'_{ij} = |\uparrow\uparrow\rangle\langle\psi_-|_{ij} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \end{pmatrix} \quad (20)$$

$$c''_{ij} = |\psi_+\rangle\langle\psi_-|_{ij} = \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (21)$$

with  $|\psi_{\pm}\rangle$  defined as in Eq. (9). This choice of jump operators ensures that the quantum master equation is  $SU(2)$ -symmetric, equally pumping the system into the three possible ground states of the ferromagnetic Heisenberg interaction. It is straightforward to check that the dark states of this set of jump operators are ferromagnetic states with all spins pointing in the same direction, but the direction itself can lie anywhere on the Bloch sphere. Finally, it is worth mentioning that it is also possible to construct the appropriate jump operators to implement the antiferromagnetic model. However, in the antiferromagnetic case the fluctuations created by the dissipation are so strong that they always drive the system towards an unpolarized state.

The second set of jump operators will now lift the  $SU(2)$  symmetry in the same way the symmetry is lifted in the equilibrium model. Here, we consider the set of jump operators given by

$$c_{ij} = \sqrt{\lambda} |\uparrow\downarrow\rangle\langle\uparrow\uparrow|_{ij} = \sqrt{\lambda} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (22)$$

$$c'_{ij} = \sqrt{\lambda} |\downarrow\uparrow\rangle\langle\uparrow\uparrow|_{ij} = \sqrt{\lambda} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (23)$$

$$c''_{ij} = \sqrt{\lambda} |\uparrow\downarrow\rangle\langle\downarrow\downarrow|_{ij} = \sqrt{\lambda} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (24)$$

$$c'''_{ij} = \sqrt{\lambda} |\downarrow\uparrow\rangle\langle\downarrow\downarrow|_{ij} = \sqrt{\lambda} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (25)$$

Crucially, these jump operators have two antiferromagnetic dark states,  $|\uparrow\downarrow\rangle$  and  $|\downarrow\uparrow\rangle$ . It is straightforward to decompose all jump operators into the required elementary interaction terms as discussed in Sec. II C.

We will now perform the variational analysis of the dissipative Heisenberg model. Most importantly, there is no Hamiltonian in the problem, hence all the quantum features of the model arise from the non-commuting quantum jump operators. Additionally, all jump operators involve two-site operators, i.e., the  $\dot{\rho}_{loc}^{(ij)}$  term in Eq. (14) is zero.

We will perform the variational analysis in two parts. First, we investigate the behavior for small values of  $\lambda$ , where we expect the existence of an  $XY$  phase, in which the  $U(1)$  symmetry is spontaneously broken. Second, we look at the limit of large  $\lambda$ , where the model is expected to realize an Ising antiferromagnet with broken  $Z_2$  symmetry. To be explicit, we always focus on three-dimensional models on a cubic lattice, as in higher dimensions the product state ansatz of Eq. (11) becomes more reliable [62]. The minimization of the variational norm is done numerically using standard nonlinear optimization techniques.

As noted before, for  $\lambda = 0$  any ferromagnetic state with all spins pointing in the same direction is an exact dark state of the quantum master equation. For any finite value of  $\lambda > 0$ , however, we find that the variational minimum is realized for  $\langle\sigma_z\rangle = 0$ , i.e., the ferromagnetic order is confined to the  $XY$  plane on the Bloch sphere, thus confirming the expectation of a  $U(1)$  phase. The spontaneous magnetization thus simply follows from the length of the spin vector on the Bloch sphere. Investigating the decay of the ferromagnetic order, we find that the  $U(1)$  phase breaks down at  $\lambda_{c1} = 1/2$ , undergoing a continuous transition, see Fig. 2. Interestingly, in contrast to the equilibrium case [90, 91], we do not find that the  $XY$  phase is immediately connected to the antiferromagnet, but instead the transition is to a disordered phase. From the variational analysis, it appears that this novel intermediate phase is a paramagnet, however, ordered phases involving a nonlocal order parameter cannot be ruled out at this point.

I will now turn to the variational analysis for large values of  $\lambda$ . In the limit  $\lambda \rightarrow \infty$ , the system has two dark states corresponding to the antiferromagnetic ordering of the spins. To incorporate antiferromagnetic ordering into the variational approach it is necessary to consider variational states of the form

$$\rho = \rho_A \otimes \rho_B \otimes \rho_A \otimes \rho_B \otimes \dots \quad (26)$$

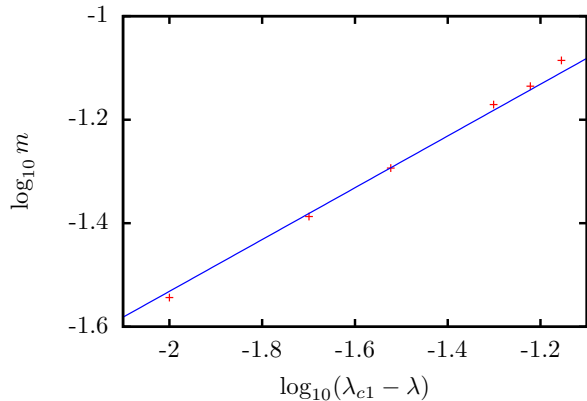


FIG. 2. Critical behavior of the dissipative phase transition close to  $\lambda_{c1} = 1/2$  between the XY and disordered phase. The fit represents a critical scaling of the order parameter according to  $m \sim (\lambda_{c1} - \lambda)^{1/2}$ .

The amount of antiferromagnetic ordering is captured in the staggered magnetization  $m_s$ , which is reduced for finite values of  $\lambda$ . Again, we find a continuous phase transition at  $\lambda_{c2} = 3/2$  to the same disordered phase, see Fig. 3. For both continuous transitions, the effective critical theory appears to be a  $\phi^4$  theory, resulting in the Landau critical exponent of  $\beta = 1/2$ . The full phase diagram of the dissipative Heisenberg model, including a comparison to the equilibrium phase diagram, is shown in Fig. 4.

The appearance of the intermediate disordered phase can be understood from the fact that the situation in open systems is quite different to finding the ground state of an equilibrium model. When constructing a variational product state solution to a ground state problem, one can never lower the variational energy by going from pure to

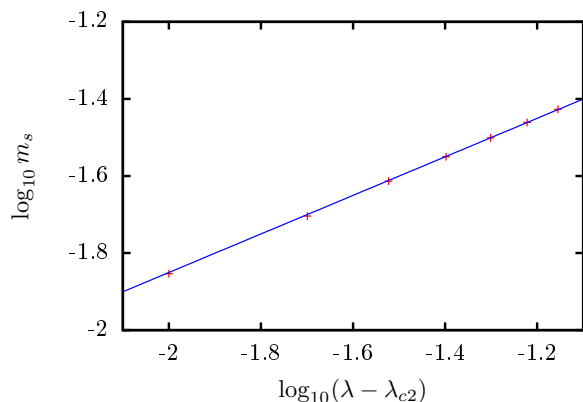


FIG. 3. Critical behavior of the dissipative phase transition close  $\lambda_{c2} = 3/2$  between the Ising antiferromagnet and the disordered phase. The fit represents a critical scaling of the order parameter indicating staggered magnetization according to  $m_s \sim (\lambda - \lambda_{c2})^{1/2}$ .

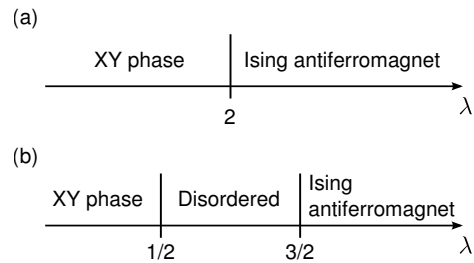


FIG. 4. Phase diagram for the Heisenberg model. (a) Ground state phase diagram exhibiting a single transition from an XY phase to an Ising antiferromagnet. (b) Phase diagram of the dissipative model showing the presence of an additional disordered phase between the XY ferromagnet and the Ising antiferromagnet.

mixed states. Hence, all variational product states are pure and can be characterized by unit vectors on the Bloch sphere. However, in open systems it is well possible that going from pure states to mixed states leads to a better approximation of the steady state. In this case, the length of the Bloch vector is reduced until it vanishes completely in a disordered phase. This is especially likely if the system is far away from an exact dark state solution, which is the case around  $\lambda \approx 1$  for the dissipative Heisenberg model. However, in contrast to the situation in [61], the fully unpolarized state is never an exact steady state of the quantum master equation, leaving the possibility for more exotic ordering mechanisms to be realized within the dissipative Heisenberg model.

#### IV. SUMMARY

I have presented a generic way to realize a large class of dissipative quantum many-body spin models, including purely dissipative model without any coherent dynamics. By performing a variational analysis of a purely dissipative Heisenberg model, I have shown that the steady state phase diagram exhibits an intermediate disordered phase that is not present in the ground state phase diagram. These findings underscore that the behavior of dissipative quantum many-body systems is potentially richer than in equilibrium, and that the recently introduced variational principle is very well suited to treat such dissipative quantum many-body problems. In future developments, it will be possible to adapt the variational principle to nonlocal variational parameters in order to analyze whether the intermediate phase is characterized by an exotic hidden order parameter.

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- [1] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, Quantum states and phases in driven open quantum systems with cold atoms, *Nature Phys.* **4**, 878 (2008).
- [2] F. Verstraete, M. M. Wolf, and J. Ignacio Cirac, Quantum computation and quantum-state engineering driven by dissipation, *Nature Phys.* **5**, 633 (2009).
- [3] H. Weimer, M. Müller, I. Lesanovsky, P. Zoller, and H. P. Büchler, A Rydberg quantum simulator, *Nature Phys.* **6**, 382 (2010).
- [4] J. T. Barreiro, M. Müller, P. Schindler, D. Nigg, T. Monz, M. Chwalla, M. Hennrich, C. F. Roos, P. Zoller, and R. Blatt, An open-system quantum simulator with trapped ions, *Nature* **470**, 486 (2011).
- [5] H. Krauter, C. A. Muschik, K. Jensen, W. Wasilewski, J. M. Petersen, J. I. Cirac, and E. S. Polzik, Entanglement Generated by Dissipation and Steady State Entanglement of Two Macroscopic Objects, *Phys. Rev. Lett.* **107**, 080503 (2011).
- [6] Y. Lin, J. P. Gaebler, F. Reiter, T. R. Tan, R. Bowler, A. S. Sørensen, D. Leibfried, and D. J. Wineland, Dissipative production of a maximally entangled steady state of two quantum bits, *Nature* **504**, 415 (2013).
- [7] S. Shankar, M. Hatridge, Z. Leghtas, K. M. Sliwa, A. Narla, U. Vool, S. M. Girvin, L. Frunzio, M. Mirrahimi, and M. H. Devoret, Autonomously stabilized entanglement between two superconducting quantum bits, *Nature* **504**, 419 (2013).
- [8] S. Diehl, W. Yi, A. J. Daley, and P. Zoller, Dissipation-Induced  $d$ -Wave Pairing of Fermionic Atoms in an Optical Lattice, *Phys. Rev. Lett.* **105**, 227001 (2010).
- [9] A. F. Alharbi and Z. Ficek, Deterministic creation of stationary entangled states by dissipation, *Phys. Rev. A* **82**, 054103 (2010).
- [10] G. Watanabe and H. Mäkelä, Dissipation-induced squeezing, *Phys. Rev. A* **85**, 023604 (2012).
- [11] C. Ates, B. Olmos, W. Li, and I. Lesanovsky, Dissipative Binding of Lattice Bosons through Distance-Selective Pair Loss, *Phys. Rev. Lett.* **109**, 233003 (2012).
- [12] D. D. B. Rao and K. Mølmer, Dark Entangled Steady States of Interacting Rydberg Atoms, *Phys. Rev. Lett.* **111**, 033606 (2013).
- [13] A. W. Carr and M. Saffman, Preparation of Entangled and Antiferromagnetic States by Dissipative Rydberg Pumping, *Phys. Rev. Lett.* **111**, 033607 (2013).
- [14] M. Lemeshko and H. Weimer, Dissipative binding of atoms by non-conservative forces, *Nature Commun.* **4**, 2230 (2013).
- [15] H. Weimer, Quantum simulation of many-body spin interactions with ultracold polar molecules, *Mol. Phys.* **111**, 1753 (2013).
- [16] S. G. Hofer, D. V. Vasilyev, M. Aspelmeyer, and K. Hammerer, Time-Continuous Bell Measurements, *Phys. Rev. Lett.* **111**, 170404 (2013).
- [17] J. Otterbach and M. Lemeshko, Dissipative Preparation of Spatial Order in Rydberg-Dressed Bose-Einstein Condensates, *Phys. Rev. Lett.* **113**, 070401 (2014).
- [18] M. J. Hartmann, Polariton Crystallization in Driven Arrays of Lossy Nonlinear Resonators, *Phys. Rev. Lett.* **104**, 113601 (2010).
- [19] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Dicke quantum phase transition with a superfluid gas in an optical cavity, *Nature* **464**, 1301 (2010).
- [20] D. Nagy, G. Kónya, G. Szirmai, and P. Domokos, Dicke-Model Phase Transition in the Quantum Motion of a Bose-Einstein Condensate in an Optical Cavity, *Phys. Rev. Lett.* **104**, 130401 (2010).
- [21] S. Diehl, A. Tomadin, A. Micheli, R. Fazio, and P. Zoller, Dynamical Phase Transitions and Instabilities in Open Atomic Many-Body Systems, *Phys. Rev. Lett.* **105**, 015702 (2010).
- [22] A. Tomadin, S. Diehl, and P. Zoller, Nonequilibrium phase diagram of a driven and dissipative many-body system, *Phys. Rev. A* **83**, 013611 (2011).
- [23] T. E. Lee, H. Häffner, and M. C. Cross, Antiferromagnetic phase transition in a nonequilibrium lattice of Rydberg atoms, *Phys. Rev. A* **84**, 031402 (2011).
- [24] E. M. Kessler, G. Giedke, A. Imamoglu, S. F. Yelin, M. D. Lukin, and J. I. Cirac, Dissipative phase transition in a central spin system, *Phys. Rev. A* **86**, 012116 (2012).
- [25] M. Hönig, M. Moos, and M. Fleischhauer, Critical exponents of steady-state phase transitions in fermionic lattice models, *Phys. Rev. A* **86**, 013606 (2012).
- [26] M. Hönig, D. Muth, D. Petrosyan, and M. Fleischhauer, Steady-state crystallization of Rydberg excitations in an optically driven lattice gas, *Phys. Rev. A* **87**, 023401 (2013).
- [27] B. Horstmann, J. I. Cirac, and G. Giedke, Noise-driven dynamics and phase transitions in fermionic systems, *Phys. Rev. A* **87**, 012108 (2013).
- [28] E. G. D. Torre, S. Diehl, M. D. Lukin, S. Sachdev, and P. Strack, Keldysh approach for nonequilibrium phase transitions in quantum optics: Beyond the Dicke model in optical cavities, *Phys. Rev. A* **87**, 023831 (2013).
- [29] L. M. Sieberer, S. D. Huber, E. Altman, and S. Diehl, Dynamical Critical Phenomena in Driven-Dissipative Systems, *Phys. Rev. Lett.* **110**, 195301 (2013).
- [30] J. Qian, L. Zhou, and W. Zhang, Quantum phases of strongly interacting Rydberg atoms in triangular lattices, *Phys. Rev. A* **87**, 063421 (2013).
- [31] T. E. Lee, S. Gopalakrishnan, and M. D. Lukin, Unconventional Magnetism via Optical Pumping of Interacting Spin Systems, *Phys. Rev. Lett.* **110**, 257204 (2013).
- [32] C. Carr, R. Ritter, C. G. Wade, C. S. Adams, and K. J. Weatherill, Nonequilibrium Phase Transition in a Dilute Rydberg Ensemble, *Phys. Rev. Lett.* **111**, 113901 (2013).
- [33] C. Joshi, F. Nissen, and J. Keeling, Quantum correlations in the one-dimensional driven dissipative XY model, *Phys. Rev. A* **88**, 063835 (2013).
- [34] N. Malossi, M. M. Valado, S. Scotto, P. Huillery, P. Pillet, D. Ciampini, E. Arimondo, and O. Morsch, Full Counting Statistics and Phase Diagram of a Dissipative Rydberg Gas, *Phys. Rev. Lett.* **113**, 023006 (2014).
- [35] M. Marcuzzi, E. Levi, S. Diehl, J. P. Garrahan, and I. Lesanovsky, Universal Nonequilibrium Properties of Dissipative Rydberg Gases, *Phys. Rev. Lett.* **113**, 210401 (2014).
- [36] N. Lang and H. P. Büchler, Exploring quantum phases by driven dissipation, *Phys. Rev. A* **92**, 012128 (2015).
- [37] J. Marino and S. Diehl, Driven Markovian Quantum Criticality, *Phys. Rev. Lett.* **116**, 070407 (2016).

- [38] M. Marcuzzi, M. Buchhold, S. Diehl, and I. Lesanovsky, Absorbing State Phase Transition with Competing Quantum and Classical Fluctuations, *Phys. Rev. Lett.* **116**, 245701 (2016).
- [39] I. Pizorn, One-dimensional Bose-Hubbard model far from equilibrium, *Phys. Rev. A* **88**, 043635 (2013).
- [40] F. W. G. Transchel, A. Milsted, and T. J. Osborne, A Monte Carlo Time-Dependent Variational Principle, *arXiv:1411.5546* (2014).
- [41] J. Cui, J. I. Cirac, and M. C. Bañuls, Variational Matrix Product Operators for the Steady State of Dissipative Quantum Systems, *Phys. Rev. Lett.* **114**, 220601 (2015).
- [42] E. Mascarenhas, H. Flayac, and V. Savona, Matrix-product-operator approach to the nonequilibrium steady state of driven-dissipative quantum arrays, *Phys. Rev. A* **92**, 022116 (2015).
- [43] J. J. Mendoza-Arenas, S. R. Clark, S. Felicetti, G. Romero, E. Solano, D. G. Angelakis, and D. Jaksch, Beyond mean-field bistability in driven-dissipative lattices: Bunching-antibunching transition and quantum simulation, *Phys. Rev. A* **93**, 023821 (2016).
- [44] A. H. Werner, D. Jaschke, P. Silvi, M. Kliesch, T. Calarco, J. Eisert, and S. Montangero, Positive Tensor Network Approach for Simulating Open Quantum Many-Body Systems, *Phys. Rev. Lett.* **116**, 237201 (2016).
- [45] A. Tomadin, V. Giovannetti, R. Fazio, D. Gerace, I. Carusotto, H. E. Türeci, and A. Imamoglu, Signatures of the superfluid-insulator phase transition in laser-driven dissipative nonlinear cavity arrays, *Phys. Rev. A* **81**, 061801 (2010).
- [46] K. Liu, L. Tan, C.-H. Lv, and W.-M. Liu, Quantum phase transition in an array of coupled dissipative cavities, *Phys. Rev. A* **83**, 063840 (2011).
- [47] T. E. Lee, H. Häffner, and M. C. Cross, Collective Quantum Jumps of Rydberg Atoms, *Phys. Rev. Lett.* **108**, 023602 (2012).
- [48] J. Jin, D. Rossini, R. Fazio, M. Leib, and M. J. Hartmann, Photon Solid Phases in Driven Arrays of Nonlinearly Coupled Cavities, *Phys. Rev. Lett.* **110**, 163605 (2013).
- [49] D. Poletti, P. Barmettler, A. Georges, and C. Kollath, Emergence of Glasslike Dynamics for Dissipative and Strongly Interacting Bosons, *Phys. Rev. Lett.* **111**, 195301 (2013).
- [50] I. Vidanović, D. Cocks, and W. Hofstetter, Dissipation through localized loss in bosonic systems with long-range interactions, *Phys. Rev. A* **89**, 053614 (2014).
- [51] M. Hoening, W. Abdussalam, M. Fleischhauer, and T. Pohl, Antiferromagnetic long-range order in dissipative Rydberg lattices, *Phys. Rev. A* **90**, 021603 (2014).
- [52] H. Weimer, Variational analysis of driven-dissipative Rydberg gases, *Phys. Rev. A* **91**, 063401 (2015).
- [53] V. R. Overbeck and H. Weimer, Time evolution of open quantum many-body systems, *Phys. Rev. A* **93**, 012106 (2016).
- [54] M. F. Maghrebi and A. V. Gorshkov, Nonequilibrium many-body steady states via Keldysh formalism, *Phys. Rev. B* **93**, 014307 (2016).
- [55] M. Eckstein, A. Hackl, S. Kehrein, M. Kollar, M. Moeckel, P. Werner, and F. A. Wolf, New theoretical approaches for correlated systems in nonequilibrium, *Eur. Phys. J. Spec. Top.* **180**, 217 (2009).
- [56] S. Finazzi, A. Le Boité, F. Storme, A. Baksic, and C. Ciuti, Corner-Space Renormalization Method for Driven-Dissipative Two-Dimensional Correlated Systems, *Phys. Rev. Lett.* **115**, 080604 (2015).
- [57] A. C. Y. Li, F. Petruccione, and J. Koch, Resummation for Nonequilibrium Perturbation Theory and Application to Open Quantum Lattices, *Phys. Rev. X* **6**, 021037 (2016).
- [58] J. Jin, A. Biella, O. Viyuela, L. Mazza, J. Keeling, R. Fazio, and D. Rossini, Cluster mean-field approach to the steady-state phase diagram of dissipative spin systems, *arXiv:1602.06553* (2016).
- [59] H. Weimer, Variational Principle for Steady States of Dissipative Quantum Many-Body Systems, *Phys. Rev. Lett.* **114**, 040402 (2015).
- [60] J. Kaczmarczyk, H. Weimer, and M. Lemeshko, Dissipative preparation of antiferromagnetic order in the Fermi-Hubbard model, *arXiv:1601.00646* (2016).
- [61] J. Lammers, H. Weimer, and K. Hammerer, Open-system many-body dynamics through interferometric measurements and feedback, *arXiv:1606.04475* (2016).
- [62] V. R. Overbeck, M. F. Maghrebi, A. V. Gorshkov, and H. Weimer, Multicritical behavior in dissipative Ising models, *arXiv:1606.08863* (2016).
- [63] T. M. Weber, M. Honing, T. Niederprum, T. Manthey, O. Thomas, V. Guarnera, M. Fleischhauer, G. Barontini, and H. Ott, Mesoscopic Rydberg-blockaded ensembles in the supratom regime and beyond, *Nature Phys.* **11**, 157 (2015).
- [64] J. Kasprzak, M. Richard, S. Kundermann, A. Baas, P. Jeambrun, J. M. J. Keeling, F. M. Marchetti, M. H. Szymaska, R. Andr, J. L. Staehli, V. Savona, P. B. Littlewood, B. Deveaud, and L. S. Dang, Bose-Einstein condensation of exciton polaritons, *Nature* **443**, 409 (2006).
- [65] C. W. Lai, N. Y. Kim, S. Utsunomiya, G. Roumpos, H. Deng, M. D. Fraser, T. Byrnes, P. Recher, N. Kumada, T. Fujisawa, and Y. Yamamoto, Coherent zero-state and  $\pi$ -state in an exciton-polariton condensate array, *Nature* **450**, 529 (2007).
- [66] A. Amo, D. Sanvitto, F. P. Laussy, D. Ballarini, E. d. Valle, M. D. Martin, A. Lematre, J. Bloch, D. N. Krizhanovskii, M. S. Skolnick, C. Tejedor, and L. Via, Collective fluid dynamics of a polariton condensate in a semiconductor microcavity, *Nature* **457**, 291 (2009).
- [67] T. C. H. Liew and V. Savona, Multimode entanglement in coupled cavity arrays, *New Journal of Physics* **15**, 025015 (2013).
- [68] A. Le Boité, G. Orso, and C. Ciuti, Steady-State Phases and Tunneling-Induced Instabilities in the Driven Dissipative Bose-Hubbard Model, *Phys. Rev. Lett.* **110**, 233601 (2013).
- [69] C. Aron, M. Kulkarni, and H. E. Türeci, Photon-Mediated Interactions: A Scalable Tool to Create and Sustain Entangled States of  $N$  Atoms, *Phys. Rev. X* **6**, 011032 (2016).
- [70] M. E. Kimchi-Schwartz, L. Martin, E. Flurin, C. Aron, M. Kulkarni, H. E. Türeci, and I. Siddiqi, Stabilizing Entanglement via Symmetry-Selective Bath Engineering in Superconducting Qubits, *Phys. Rev. Lett.* **116**, 240503 (2016).
- [71] L. Robledo, L. Childress, H. Bernien, B. Hensen, P. F. A. Alkemade, and R. Hanson, High-fidelity projective readout of a solid-state spin quantum register, *Nature* **477**, 574 (2011).
- [72] N. Bar-Gill, L. M. Pham, C. Belthangady, D. Le Sage, P. Cappellaro, J. R. Maze, M. D. Lukin, A. Yacoby, and



- R. Walsworth, Suppression of spin-bath dynamics for improved coherence of multi-spin-qubit systems, *Nature Commun.* **3**, 858 (2012).
- [73] H. Weimer, N. Y. Yao, C. R. Laumann, and M. D. Lukin, Long-Range Quantum Gates using Dipolar Crystals, *Phys. Rev. Lett.* **108**, 100501 (2012).
- [74] H. Weimer, N. Y. Yao, and M. D. Lukin, Collectively Enhanced Interactions in Solid-State Spin Qubits, *Phys. Rev. Lett.* **110**, 067601 (2013).
- [75] I. de Vega and D. Alonso, Dynamics of non-Markovian open quantum systems, *Rev. Mod. Phys.* (to be published), arXiv:1511.06994.
- [76] J. Iles-Smith, A. G. Dijkstra, N. Lambert, and A. Nazir, Energy transfer in structured and unstructured environments: Master equations beyond the Born-Markov approximations, *J. Chem. Phys.* **144** (2016).
- [77] H. Weimer, M. Müller, H. P. Büchler, and I. Lesanovsky, Digital quantum simulation with Rydberg atoms, *Quant. Inf. Proc.* **10**, 885 (2011).
- [78] I. M. Georgescu, S. Ashhab, and F. Nori, Quantum simulation, *Rev. Mod. Phys.* **86**, 153 (2014).
- [79] F. Reiter and A. S. Sørensen, Effective operator formalism for open quantum systems, *Phys. Rev. A* **85**, 032111 (2012).
- [80] A. W. Glaetzle, M. Dalmonte, R. Nath, C. Gross, I. Bloch, and P. Zoller, Designing Frustrated Quantum Magnets with Laser-Dressed Rydberg Atoms, *Phys. Rev. Lett.* **114**, 173002 (2015).
- [81] R. M. W. van Bijnen and T. Pohl, Quantum Magnetism and Topological Ordering via Rydberg Dressing near Förster Resonances, *Phys. Rev. Lett.* **114**, 243002 (2015).
- [82] A. Micheli, G. K. Brennen, and P. Zoller, A toolbox for lattice-spin models with polar molecules, *Nature Phys.* **2**, 341 (2006).
- [83] A. V. Gorshkov, S. R. Manmana, G. Chen, J. Ye, E. Demler, M. D. Lukin, and A. M. Rey, Tunable Superfluidity and Quantum Magnetism with Ultracold Polar Molecules, *Phys. Rev. Lett.* **107**, 115301 (2011).
- [84] A. V. Gorshkov, K. R. A. Hazzard, and A. M. Rey, Kitaev honeycomb and other exotic spin models with polar molecules, arXiv:1301.5636 (2013).
- [85] J. Honer, H. Weimer, T. Pfau, and H. P. Büchler, Collective Many-Body Interaction in Rydberg Dressed Atoms, *Phys. Rev. Lett.* **105**, 160404 (2010).
- [86] H. P. Büchler, A. Micheli, and P. Zoller, Three-body interactions with cold polar molecules, *Nature Phys.* **3**, 726 (2007).
- [87] T. E. Lee, Y. N. Joglekar, and P. Richerme, String order via Floquet interactions in atomic systems, arXiv:1605.05738 (2016).
- [88] H. Weimer, M. J. Henrich, F. Rempp, H. Schröder, and G. Mahler, Local effective dynamics of quantum systems: A generalized approach to work and heat, *Europhys. Lett.* **83**, 30008 (2008).
- [89] C. N. Yang and C. P. Yang, Ground-State Energy of a Heisenberg-Ising Lattice, *Phys. Rev.* **147**, 303 (1966).
- [90] H.-J. Mikeska and A. K. Kolezhuk, in *Quantum Magnetism*, edited by U. Schollwöck, J. Richter, D. J. J. Farnell, and R. F. Bishop (Springer, Berlin, 2004).
- [91] Y. Fukumoto, Thermodynamic Properties of the Square Lattice XXZ Model: A Schwinger Boson Mean-Field Theory, *J. Phys. Soc. Jpn.* **65**, 569 (1996).